Exercise 17

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$y = \int_{\sqrt{x}}^{\pi/4} \theta \tan\theta \, d\theta$$

Solution

According to part 1 of the fundamental theorem of calculus,

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

Switch the limits of integration to put the variable part on top.

$$y = -\int_{\pi/4}^{\sqrt{x}} \theta \tan\theta \, d\theta$$

In order to make the upper limit a single variable, let $u = \sqrt{x}$.

$$y = -\int_{\pi/4}^{u} \theta \tan\theta \, d\theta$$

As a result, using the chain rule,

$$\begin{aligned} y' &= -\frac{d}{dx}\int_{\pi/4}^{u} \theta \tan\theta \, d\theta \\ &= -\frac{du}{dx}\frac{d}{du}\int_{\pi/4}^{u} \theta \tan\theta \, d\theta \\ &= -\frac{du}{dx}(u\tan u) \\ &= -\frac{1}{2}x^{-1/2}\left(\sqrt{x}\tan\sqrt{x}\right) \\ &= -\frac{1}{2}\tan\sqrt{x}. \end{aligned}$$